

Unified Maximally Natural Supersymmetry

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ABSTRACT: Maximally Natural Supersymmetry, an unusual weak-scale supersymmetric extension of the Standard Model based upon the inherently higher-dimensional mechanism of Scherk-Schwarz supersymmetry breaking (SSSB), possesses remarkably good fine tuning given present LHC limits. Here we construct a version with precision $SU(2)_L \times U(1)_Y$ unification: $\sin^2 \theta_W(M_Z) \simeq 0.231$ is predicted to $\pm 2\%$ by unifying $SU(2)_L \times U(1)_Y$ into a 5D $SU(3)_{EW}$ theory at a Kaluza-Klein scale of $1/R_5 \sim 4.4$ TeV, where SSSB is simultaneously realised. Full unification with $SU(3)_C$ is accommodated by extending the 5D theory to a $N = 4$ supersymmetric $SU(6)$ gauge theory on a 6D rectangular orbifold at $1/R_6 \sim 40$ TeV. TeV-scale states beyond the SM include exotic charged fermions implied by $SU(3)_{EW}$ with masses lighter than ~ 1.2 TeV, and squarks in the mass range 1.4 TeV – 2.3 TeV, providing distinct signatures and discovery opportunities for LHC run II.

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1 Introduction

Recently [1–3] there has been renewed interest in supersymmetric (SUSY) models that resolve the little hierarchy problem of the Standard Model [4–9] by utilizing the remarkable properties of the Scherk-Schwarz supersymmetry breaking (SSSB) mechanism [10, 11]. The SSSB mechanism, previously applied to weak-scale SUSY model building by a variety of authors [12–25], breaks SUSY by boundary conditions (bc’s) in one (or more) extra dimensions. The new implementation of this idea, so-called Maximally Natural Supersymmetry (MNSUSY) [1–3] is consistent with LHC run I constraints on superpartners, rare process and flavor constraints, and achieves the observed higgs mass, while maintaining a low fine tuning of $\sim 20\%$. This improvement in tuning compared to MSSM-like SUSY models mostly follows from the fact that SSSB locally maintains unbroken SUSY at each point of the extra dimension, with SUSY only being broken non-locally with respect to the extra dimension. This non-locality of SUSY breaking then protects the higgs boson from getting large log-enhanced loop corrections to its soft mass squared [26–29]. For maximal SSSB of an underlying 5D SUSY theory, the case utilized in the MNSUSY construction, the extra dimension may equivalently be thought of as an $S^1/(Z_2 \times Z'_2)$ orbifold with boundary fixed points, and we will use this description in the rest of this work.¹

However, the SSSB-based models studied so far, including MNSUSY, give up one of the most attractive successes of MSSM-like theories, namely precision gauge coupling unification and the associated prediction of the weak mixing angle $\sin^2 \theta_W(M_Z)$ [39, 40]. In conventional SUSY theories, the unification of couplings is realised with differential logarithmic running of Standard Model gauge couplings up to a high scale, $\sim 2 \times 10^{16}$ GeV, but this is not possible with extra dimensions at the TeV scale and an associated cutoff much below 10^{16} GeV.

On the other hand, the relative closeness between the observed $\sin^2 \theta_W^{\text{MS}}(M_Z) \approx 0.23116 \pm 0.00012$, and $1/4$ motivate consideration of models where $SU(2)_L \times U(1)_Y$ is unified *at low scales* into an $SU(3)_{\text{EW}}$ gauge group [41–44] where $\sin^2 \theta_W = 1/4$ is the unified prediction before logarithmic radiative corrections. This, of course, is only a partial unification, but in principle $SU(3)_{\text{EW}}$ can be unified with $SU(3)_C$ into a simple group in 6D [45–48]. In this work, we construct a unified 6D model where the effective theory at low energies is MNSUSY (with some specified exotic states), and we study its predictions and low-energy phenomenology.

A schematic illustration of our 6D model is shown in Figure 1 (the possible further embedding in a yet higher-dimensional gravitational bulk is shown in the right panel of Figure 1). A $N = 4$ supersymmetric² $SU(6)$ gauge theory is compactified on a $S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)$ orbifold with symmetry structure as shown in Figure 1 [45, 46]. The leptons

¹Gauge symmetry breaking by orbifold bcs, also known as the Hosotani mechanism [30–32], is an important ingredient in constructing simple, realistic grand unified theories (GUTs), as it gives an elegant solution to the doublet triplet splitting problem [33–35] while maintaining precision SUSY gauge-coupling unification [35–38].

²Throughout this work the size of the supersymmetry algebra is expressed in terms of the corresponding supersymmetry algebra in four dimensions.

and higgses, now extended to full $SU(3)_{EW}$ triplets, ($\mathcal{L} = \{L, e^c\}$, $\mathcal{H}_{u,d} = \{H_{u,d}, S_{u,d}\}$), are localized on the (4+1)-dimensional brane at $z = 0$ preserving $SU(3)_C \times SU(3)_{EW} \times U(1)'$ and $N = 2$ SUSY. In distinction, the quark super-multiplets Q, u^c, d^c cannot be embedded in $SU(3)_{EW}$, and must be localized on the (3+1)-dimensional orbifold fixed point at $y = \pi R_5, z = 0$ where just the SM gauge group and $N = 1$ SUSY are good symmetries.

The benchmark size of the fifth dimension is chosen to be $1/R_5 \sim 4.4$ TeV to be compatible with experiment and at the same time minimize the fine-tuning, while R_6 is only constrained by the unitarity bound on gauge boson scattering in higher-dimensional gauge theories, as we discuss in Section 2.3. In the benchmark model, we have an effective GUT scale $M_{GUT} \sim 10(1/\pi R_6) \sim 90(1/\pi R_5) \sim 100$ TeV. (Throughout the paper, we work in the limit $M_{GUT}, 1/R_6 \gg 1/R_5$.)

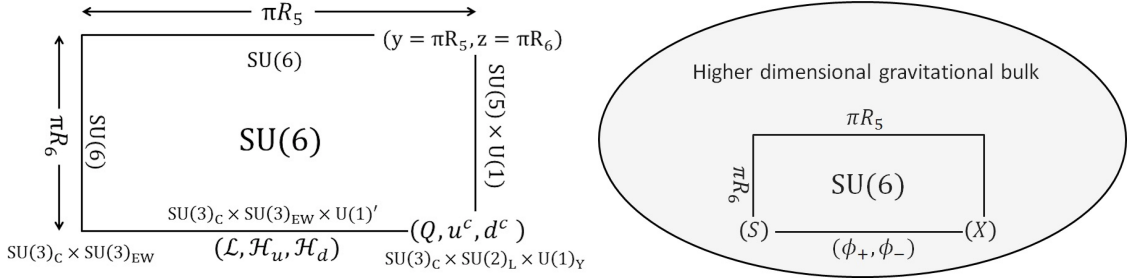


Figure 1. *Left panel:* Schematic illustration of the symmetry structure of the minimal $SU(6)$ unified model. In this plot, each line is a (4+1)-dimensional orbifold line and each corner is a (3+1)-dimensional orbifold fixed point in the two extra dimensions (y, z) of the total 6D space. The gauge symmetry on each brane is as indicated, or the unbroken $SU(6)$ otherwise. The bulk $N = 4$ supersymmetric $SU(6)$ gauge symmetry is broken by orbifold bcs. The gauge symmetry on the bottom line ($z = 0$ brane) is $SU(3)_C \times SU(3)_{EW} \times U(1)'$. The leptons ($\mathcal{L} = (L, e^c)$) and higgses ($\mathcal{H}_{u,d} = (H_{u,d}, S_{u,d})$) are simultaneously 5D SUSY hyper-multiplets and triplets respecting the $SU(3)_{EW}$ gauge symmetry. The quark chiral multiplets Q, u^c, d^c do not respect the $SU(3)_{EW}$ gauge symmetry and therefore live on the $y = \pi R_5, z = 0$ brane where the gauge symmetry is broken to the SM gauge group. Different, incompatible $N = 1$ SUSYs are realized on the $y = \pi R_5, z = 0$ and $y = 0, z = 0$ branes, resulting in non-local SUSY-breaking on the 5 dimensional interval. *Right panel:* The embedding of the 6D theory into a higher dimensional gravitational bulk with Standard Model singlets indicated. These singlet states are required to break the $U(1)'$ gauge symmetry and generate masses for the light modes.

$SU(2)_L \times U(1)_Y$ unifies into $SU(3)_{EW}$ due to differential logarithmic running from M_Z to M_{GUT} [41–44]. The beta functions can be calculated in the full energy range and calculable $SU(3)_{EW}$ breaking threshold corrections at $1/R_5$ can be obtained by the method in [49]. On the other hand, the non-calculable $SU(3)_{EW}$ breaking threshold corrections are estimated using the strong coupling analysis [50]. The scale where $SU(2)_L \times U(1)_Y$ unifies into $SU(3)_{EW}$ depends on only physics below or around $1/R_5$, namely, the squark masses, the heavy higgs masses and masses of gauginos and singlet higgs zero modes. A detailed discussion of this $SU(3)_{EW}$ unification is contained in Section 2.

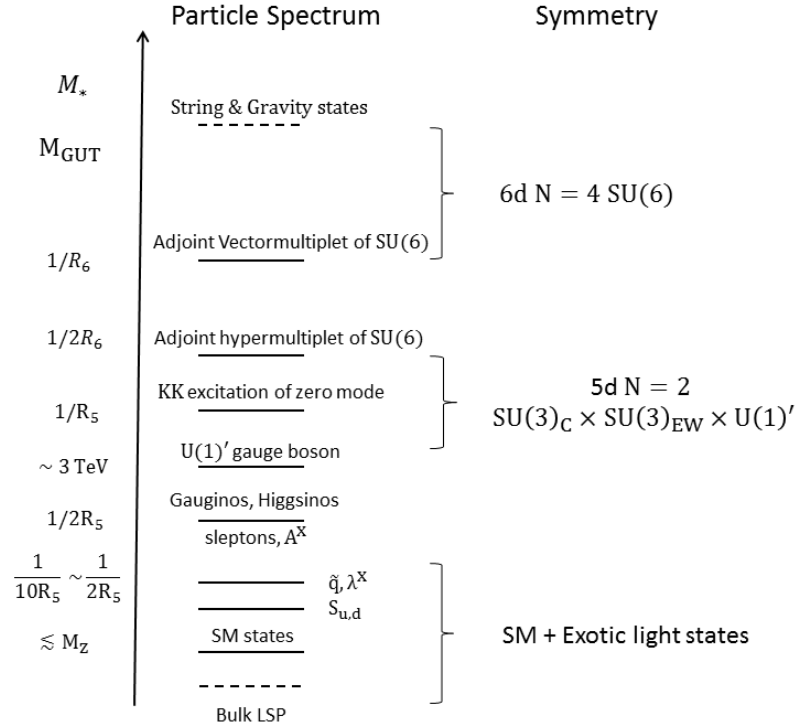


Figure 2. Schematic spectrum of states that are of experimental interest, and states that are required so as to get $SU(6)$ unification. The symmetry of the two stage unification model at various energy scales is shown on the right of the plot. The energy axis is not to scale.

Moving on to the more ambitious objective of a full unification of $SU(3)_{EW} \times SU(3)_C$ into a single, higher dimensional GUT structure, we utilize recent knowledge of extended SUSY theories in higher dimensions [51–54]. In particular, by studying the properties of the pre-potential, it has been shown that higher-dimensional operators involving gauge fields at two derivative order are highly restricted by $N = 2$ SUSY and gauge symmetry, and that power-law threshold corrections can be, in some cases, exactly calculated [55, 56]. These analyses lay the foundation of our study of precision unification by power-law threshold corrections.

Specifically, we argue that $SU(3)_C \times SU(3)_{EW}$ unifies into $SU(6)$ on a 6D orbifold due to differential logarithmic running from M_Z to M_{GUT} and, dominantly, calculable power (linear) threshold corrections [55, 56], the differential linear threshold corrections being due to the non- $SU(6)$ symmetric matter content on the $z = 0$ brane. The scale where $SU(3)_C \times SU(3)_{EW}$ unifies into $SU(6)$ depends on dominantly the size, πR_5 , of the $z = 0$ brane together with its particle content. A detailed discussion of this $SU(6)$ unification is in Section 3.

The $SU(6)$ unifying group also contains, after breaking by orbifold bcs, a surviving $U(1)'$ gauge symmetry in addition to the SM. In fact such a symmetry was posited in the original MNSUSY model [1, 3] so as to raise the physical higgs mass to its observed value. $U(1)'$ is

broken by vacuum expectation values (vevs) of the 5D fields, ϕ_{\pm} , and a contribution from the non-decoupling D-term [57] of the broken $U(1)'$ lifts m_h to 125 GeV. A detail account of this dynamics in our context can be found in Section 4.

Finally, in Section 5 we discuss the collider phenomenology of our unified model, focussing in particular on the aspects that differ from the basic MNSUSY model, and also on issues directly related to unification. Interestingly we find that the broken $U(1)'$, together with $U(1)_R$ symmetry and locality forbid the dangerous proton decay and neutron anti-neutron oscillation operators (Section 5.4).

2 $SU(3)_{\text{EW}}$ unification

In this section, we discuss how $SU(3)_{\text{EW}}$ unification can be realized with one TeV-sized dimension in a model that simultaneously realizes the MNSUSY scenario at the TeV-scale. In later sections we will go on to show how this 5D $N = 2$ SUSY theory can be extended to a full $SU(6)$ $N = 4$ SUSY unification in 6D. Our 5D model is based upon the non-supersymmetric Dimopoulos-Kaplan $SU(3)_{\text{EW}}$ model [41–44], but we extend their construction to realize SSSB, and perform a precision calculation of the prediction for $\sin^2 \theta_W$.

2.1 Minimal matter content and orbifold boundary conditions

Consider a 5D $N = 2$ SUSY $SU(3)_{\text{EW}}$ gauge theory compactified on a $S^1/Z_2 \times Z'_2$ orbifold line segment (where $y \in [0, \pi R_5]$) with bc's on the $SU(3)_{\text{EW}}$ gauge fields $A_\mu \equiv A_\mu^a T^a$ determined by the orbifold equivalences

$$\begin{aligned} A_\mu(x^\mu, y) &= A_\mu(x^\mu, -y) \quad \text{and} \\ A_\mu(x^\mu, y) &= Z' A_\mu(x^\mu, -y + 2\pi R_5) Z'^{-1} \end{aligned} \tag{2.1}$$

where $Z' = \text{diag}(1, 1, -1)$ acts on $SU(3)_{\text{EW}}$ indices, and the action on the A_5 components has an extra overall minus sign in both relations. We need to extend this action to the full field content of the theory. As minimal 5D bulk SUSY corresponds, from a 4D perspective, to $N = 2$ SUSY. Let us first recall how $N = 2$ SUSY is written in terms of 4D $N = 1$ supermultiplets: The vector supermultiplet of $N = 2$ contains a $N = 1$ vector supermultiplet, V , and a chiral supermultiplet, $\tilde{\Sigma}$, both adjoint representation fields. In terms of on-shell degrees of freedom $V = (A_\mu, \lambda)$ and $\Sigma = (\tilde{\Sigma}, \psi)$, where λ and ψ are Weyl fermions (the complex scalar field $\tilde{\Sigma}$ contains A_5). Similarly, the $N = 2$ hypermultiplet contains two 4D $N = 1$ chiral supermultiplets with conjugate gauge quantum numbers and R-charge.

These bc's under (Z_2, Z'_2) are extended to the component fields of the full vector multiplet

$$\begin{aligned}
A_\mu : & \begin{pmatrix} (+, +) & (+, +) & (+, -) \\ (+, +) & (+, +) & (+, -) \\ (+, -) & (+, -) & (+, +) \end{pmatrix} & \tilde{\Sigma} : & \begin{pmatrix} (-, -) & (-, -) & (-, +) \\ (-, -) & (-, -) & (-, +) \\ (-, +) & (-, +) & (-, -) \end{pmatrix} \\
\lambda : & \begin{pmatrix} (+, -) & (+, -) & (+, +) \\ (+, -) & (+, -) & (+, +) \\ (+, +) & (+, +) & (+, -) \end{pmatrix} & \psi : & \begin{pmatrix} (-, +) & (-, +) & (-, -) \\ (-, +) & (-, +) & (-, -) \\ (-, -) & (-, -) & (-, +) \end{pmatrix}, \quad (2.2)
\end{aligned}$$

where we have specified the bc's that apply to the various $SU(3)$ components, with $+$ and $-$ corresponding to Neumann and Dirichlet respectively. This action breaks $SU(3)_{\text{EW}}$ down to $SU(2)_L \times U(1)_Y$ on the 4D orbifold brane at $y = \pi R_5$. Moreover, the bc's of gauginos and scalars are such that *two different* $N = 1$ SUSYs are preserved locally on each boundary, while the incompatibility of the two $N = 1$ SUSYs breaks SUSY completely and non-locally by (maximal) SSSB. The $(+, +)$ zero modes in this sector of the theory include the $SU(2)_L \times U(1)_Y$ gauge bosons A_μ^{EW} and the gauginos λ^X . The KK tower for the vector multiplet contains the modes with $(+, +)$ and $(-, -)$ bc's at mass $(n + 1)/R_5$, and the modes with $(+, -)$ or $(-, +)$ bc's at mass $(n + 1/2)/R_5$.

Since the quark chiral supermultiplets, Q, u_R and d_R , have hypercharge assignments that do not descend from $SU(3)$ representations, they have to be localized on the $y = \pi R_5$ brane. On the other hand, the higgs $H_{u,d}$ and leptons L, e_R (of all 3 generations) can, in principle, be either bulk or brane localized fields consistent with $SU(3)_{\text{EW}}$. The analysis of the MNSUSY model in [1] shows that if the higgs is a bulk field then a low-fine-tuning solution to the little hierarchy problem is possible by virtue of the SSSB bc's. Thus we extend the higgs multiplets to full $SU(3)_{\text{EW}}$ triplets by the addition of fields $S_{u,d}$, giving triplets $\mathcal{H}_{u,d} \equiv (H_{u,d}, S_{u,d})$. In this way the higgs fields consistently transform under the 5D bulk $SU(3)_{\text{EW}}$ gauge symmetry. Similarly, we choose here to place the leptons also in the bulk as triplets $\mathcal{L} \equiv (L, e_R)$ – this allows suitable lepton Yukawas to be generated (See equation B.1). The orbifold bc's for the scalar and fermion components of the resulting higgs and lepton hypermultiplets are

$$\begin{aligned}
\mathcal{H}_{u,d} : & \begin{pmatrix} (+, +) \\ (+, +) \\ (+, +) \end{pmatrix}, & \mathcal{H}_{u,d}^c : & \begin{pmatrix} (-, -) \\ (-, -) \\ (-, -) \end{pmatrix}, & \tilde{\mathcal{H}}_{u,d} : & \begin{pmatrix} (+, -) \\ (+, -) \\ (+, -) \end{pmatrix}, & \tilde{\mathcal{H}}_{u,d}^c : & \begin{pmatrix} (-, +) \\ (-, +) \\ (-, +) \end{pmatrix}. \\
\begin{pmatrix} L \\ e_R \end{pmatrix} : & \begin{pmatrix} (+, +) \\ (+, +) \\ (+, +) \end{pmatrix}, & \begin{pmatrix} L^c \\ e_R^c \end{pmatrix} : & \begin{pmatrix} (-, -) \\ (-, -) \\ (-, -) \end{pmatrix}, & \begin{pmatrix} \tilde{L} \\ \tilde{e}_R \end{pmatrix} : & \begin{pmatrix} (+, -) \\ (+, -) \\ (+, -) \end{pmatrix}, & \begin{pmatrix} \tilde{L}^c \\ \tilde{e}_R^c \end{pmatrix} : & \begin{pmatrix} (-, +) \\ (-, +) \\ (-, +) \end{pmatrix}. \quad (2.3)
\end{aligned}$$

With the above set of fields and bc's, the low energy effective theory well below the compactification scale $1/R_5$ includes all fields in the SM charged under the SM electroweak symmetry, the superpartners of the brane localized quark fields, a vector pair of zero mode gauginos λ^X with quantum number $(2, \pm 3/2)$ under the $SU(2)_L \times U(1)_Y$ symmetry and a vector pair of scalars $S_{u,d}$ with quantum numbers $(1, \pm 1)$.

2.2 Differential logarithmic running in SUSY orbifold theories

Precision unification into $SU(3)_{\text{EW}}$ involves the relative logarithmic running of g_1 and g_2 . We will briefly outline the argument [49] that shows that this relative running of the gauge couplings in a 5D SUSY orbifold theory can be calculated to high precision.³

The first step is to write down all possible bulk and brane localized counterterms. The counterterm corresponding to the renormalization of the (dimensionful) 5D bulk gauge coupling has dimension $[mass]^1$ and therefore can, in principle, get both a linear and a logarithmic dependence on the UV cut-off, Λ , of the form $\Lambda + m \log \Lambda$, where m is a mass parameter in the bulk Lagrangian. In our case the linear piece is *universal* since the 5D bulk is $SU(3)_{\text{EW}}$ symmetric while the logarithmic piece vanishes due to the absence of bulk mass parameters in our theory. However, there is still coupling running above the compactification scale $1/R_5$ due to the existence of the orbifold fixed points where counterterms in the form of

$$\mathcal{L}_{ct} = \int d^4x dy F_{\mu\nu} F^{\mu\nu} [a \delta(y) + b \delta(y - \pi R_5)] \quad (2.4)$$

can be written down for a $S^1/Z_2 \times Z'_2$ orbifold, where a and b are constants that cancel the UV divergences of the orbifold gauge theory. These counterterms in 5D can be Fourier decomposed in terms of the 4D KK gauge boson excitations

$$\begin{aligned} \mathcal{L}_{ct} = \int d^4x \left\{ (a+b) \left[\sum_{n=0}^{\infty} c_{2n}^2 F_{\mu\nu}^{(2n)} F^{(2n)\mu\nu} + \sum_{n=0, k=1}^{\infty} 2c_{2n} c_{2n+4k} F_{\mu\nu}^{(2n)} F^{(2n+4k)\mu\nu} \right] \right. \\ \left. + 2(a-b) \sum_{n=0, k=1}^{\infty} c_{2n} c_{2n+4k-2} F_{\mu\nu}^{(2n)} F^{(2n+4k-2)\mu\nu} \right\}. \quad (2.5) \end{aligned}$$

Next, from examination of the 5D KK momenta in loop diagrams (including the relevant KK mode identifications due to the orbifold bc's), it follows that the exchange of odd KK modes only produces transitions $2n \rightarrow 2n + 4k - 2$ while the exchange of even KK modes only produces transitions $2n \rightarrow 2n + 4k$. Comparing with Eq.(2.5) then implies that loop diagrams involving odd KK modes lead to $a = -b$, while even KK modes in loops lead to $a = b$. Since the gauge kinetic term of the *zero mode* is not renormalized when $a = -b$, the odd KK modes do not affect the running of the effective 4D gauge coupling.

The beta function of the whole KK tower compared with that arising from the corresponding zero mode can then be calculated and understood in the following manner [27, 35, 38, 49]. The absence of logarithmic running in the absence of the orbifold fixed points implies that the sum of the beta functions due to the exchange of the zero mode and the KK modes altogether is zero if the bulk is an S^1 . Moreover, in the case of S^1 , KK modes with masses equal to the even modes and the odd modes with respect to the $S^1/Z_2 \times Z'_2$ orbifold have the *same* contribution to the beta function coefficient. Therefore, on S^1 both the even and

³Although the effect of any brane localized charged matter is not included in the discussion below, it can be accounted by purely standard 4D calculations, as we will do when we present final results.

odd modes contribute $-1/2$ to the gauge coupling running compared with the corresponding zero mode. Since, as we previously argued, in a $S^1/Z_2 \times Z'_2$ orbifold theory, the exchange of the odd modes does not contribute to the logarithmic running of gauge coupling, we finally get that *the logarithmic running of gauge coupling due to bulk states slows down above the compactification scale to half the original rate* (due to the zero plus even modes), while the running due to the brane localized matter is not affected. This concludes our brief discussion of the beta-function coefficients in a $S^1/Z_2 \times Z'_2$ orbifold field theory.

The other crucial point for precision unification is the matching between the 5D theory and the 4D effective theory below $1/R_5$. The matching can be accounted for by the following equation where the scale μ is the matching scale [49]:

$$\frac{1}{g_i^2(\mu)} = \frac{\pi R_5}{g_5^2} + \Delta_i(\mu) + \lambda_i(\mu R_5). \quad (2.6)$$

Here $\Delta_i(\mu)$ accounts for the effect of 4D gauge kinetic operators on the orbifold fixed points, and the matching function $\lambda_i(\mu R)$ encodes the radiative corrections from the massive KK modes once they are integrated out. Since the relative running is purely logarithmic, dimensional regularization is sufficient to get the form of the functions $\Delta_i(\mu)$ and $\lambda_i(\mu R)$. Up to non-calculable contributions (to which we later return), we have [49]

$$\begin{aligned} \lambda_i(\mu R) &= \frac{b_{\text{even}}}{8\pi^2} (\mathcal{I} - 1 - \log[\pi] - \log[\mu R]) + \frac{b_{\text{odd}}}{8\pi^2} (-\log[2]) \\ \Delta_i(\mu) &= \Delta_i(\Lambda) - \frac{b_{\text{even}} + b_{\text{zero}}}{8\pi^2} \log[\mu/\Lambda] \end{aligned} \quad (2.7)$$

where b_{even} and b_{odd} are the beta functions of the even and odd modes respectively, while the constant $\mathcal{I} \simeq 0.02$ has been calculated numerically.

With the above considerations, we can obtain the relative logarithmic running of gauge couplings as a function of the energy scale μ using dimensional regularization [49]

$$\begin{aligned} \frac{4\pi}{g^2(\mu)} &= \frac{1}{\alpha_{M_Z}} - \frac{b_{\text{SM}}}{2\pi} \log\left[\frac{\mu}{M_Z}\right] - \frac{b_{\text{Squark}}}{2\pi} \log\left[\frac{\mu}{M_{\text{Squark}}}\right] \\ &\quad - \frac{b_{\text{HeavyHiggs}}}{2\pi} \log\left[\frac{\mu}{M_{\text{HeavyHiggs}}}\right] - \frac{b_{\lambda^X}}{2\pi} \log\left[\frac{\mu}{M_{\lambda^X}}\right] - \frac{b_S}{2\pi} \log\left[\frac{\mu}{M_S}\right] \\ &\quad - \frac{b_{\text{even}}}{2\pi} (\log[\mu/M_R] + 1 + \log[\pi] - 0.02) - \frac{b_{\text{odd}}}{2\pi} \log[2] + \dots \end{aligned} \quad (2.8)$$

Here b_{SM} is the beta function of the SM fields, b_{Squark} and $b_{\text{HeavyHiggs}}$ are the contributions of the superpartners of the brane localized matter and the heavy higgses, respectively, b_{λ^X} and b_S are the contributions of the exotic fermion and scalar zero modes, and b_{even} and b_{odd} are the beta functions of the even and odd KK modes. We assume all three generation squarks have an almost degenerate mass spectrum $m_{\tilde{t}} \simeq m_{\tilde{q}_{1,2}}$ in the paper for phenomenological reasons (See Section 5.1 for more detailed discussions). In the above equation, we have added the running due to brane localized matter fields, which is the same as the corresponding MSSM fields in 4D. For our benchmark model, the final beta functions coefficients are summarized in Table 1.

Group	b_{SM}	b_{Squark}	$b_{\text{HeavyHiggs}}$	b_{λ^X}	b_S	b_{even}	b_{odd}
$SU(3)_C$	-7	2	0	0	0	5	-2
$SU(2)_L$	-19/6	3/2	1/6	2/3	0	4/3	-5/6
$U(1)_Y$	41/18	11/18	1/18	2	2/9	-10/3	23/6

Table 1. Beta function coefficients of the Standard Model fields, squarks, heavy higgses, zero mode charged fermion and scalars and even and odd KK modes.

2.3 Unitarity constraint on the cutoff

We now briefly turn to the issue of the UV scale at which a perturbative treatment must inevitably break down. As is well known, the scattering amplitude of gauge bosons diverges at high energies in extra dimensional theories, which implies that the theory is an effective theory which must be cut-off at a scale M_* at or below the unitarity bound Λ . The tree level unitarity bound can be found by using either NDA estimates [50] or by explicitly doing the partial wave calculation of gauge boson scattering at tree level [58–60]. The scattering amplitude can be calculated as a function of the maximum number of KK modes both in momentum space [60] and mixed position/momentum space [61]. Performing such a calculation we find that the cut-off of the theory, Λ , equal, in units of $1/R_5$, to *twice* the maximum number of KK modes (‘twice’ as this sets the effective \sqrt{s} at which the perturbative calculation of partial wave unitarity fails) allowed by partial wave unitarity is

$$\Lambda \simeq \frac{64\pi}{5N_c g_4^2(\Lambda)} \frac{1}{R_5} \quad (2.9)$$

in the limit that $\Lambda R_5 \gg 1$. Note, however, that the gauge coupling at the cut-off scale, which the dominant contribution to the scattering amplitude depends on, is quite different from the gauge coupling measured at the weak scale: The linear threshold effect between $1/R_5$ and Λ makes $g_4^2(\Lambda)$ smaller than the size at $1/R_5$, and this is especially the case for the $SU(3)_C$ gauge coupling. We include this effect when numerically evaluating the cutoff of our theory.

2.4 $\sin^2 \theta_W$ and the precision of $SU(3)_{\text{EW}}$ unification

As normal for orbifold GUTs the precision of the low-energy prediction for $\sin^2 \theta_W$ is limited by the lack of knowledge of the coefficients of the brane-localized gauge kinetic terms that do not respect the bulk unified symmetry [36–38]. This intrinsic uncertainty is typically of the same size as the unknown UV threshold corrections in traditional two-loop analyses of MSSM unification, and so is not particularly problematic for the success of unification.

Utilizing the standard strong-coupling naive dimensional analysis (NDA) [38, 50] this intrinsic uncertainty in the unification prediction due to the brane-localized kinetic terms can be estimated. Assuming that the theory is strongly coupled at the fundamental scale $M_* \leq \Lambda$ for both the unified bulk gauge kinetic term and the independent brane localized gauge kinetic terms, we can obtain the size of the corresponding coupling at the matching

scale $1/R_5$, which combines to produce the size of the gauge coupling in the 4D effective theory.

$$\frac{1}{g_i^2(1/R_5)} = \frac{\pi R_5}{g_5^2(1/R_5)} + \frac{1}{g_4^2(1/R_5)} \quad (2.10)$$

The brane localized gauge coupling $g_4^2(1/R_5)$ runs only logarithmically. With the strong coupling scale Λ not far above the compactification scale $1/R_5$, the log running does not alter the size of the overall gauge coupling as much as the linear threshold effect, especially, we expect $g_4^2(1/R_5) \simeq g_4^2(\Lambda) \sim 16\pi^2/N_c$. The bulk gauge coupling, g_5 , however, runs linearly down from the scale where the theory is unified, close to the unitarity bound, to the matching scale $1/R_5$ where the combined 4D gauge coupling $g_i^2(1/R_5)$ needs to be close to its weak scale value. A sizeable bulk will ensure that such a sizeable difference can be generated [36–38].

Numerically evaluating the relative logarithmic running and threshold effects following from Eq.(2.8) we find that the central observed value of $\sin^2 \theta_W^{\overline{\text{MS}}}(M_Z) \simeq 0.231$ is achieved when $1/R_5 \simeq 4.4 \text{ TeV}$ for our benchmark $SU(3)_{\text{EW}}$ model and matter content as discussed in Section 2.1 (with corresponding $SU(2)_L \times U(1)_Y$ beta function coefficients as displayed in Table 1). Importantly, this value of $1/R_5$ melds nicely with that required from low-fine tuning of the EW scale in MNSUSY models [1, 3]: If the value of $1/R_5$ preferred by $SU(2)_L \times U(1)_Y$ unification had been lower than $\simeq 4 \text{ TeV}$ or much higher than this scale then either the model would have been ruled out by LHC constraints on sparticle masses, or it would have been highly tuned. The uncertainty in the value of $\sin^2 \theta_W^{\overline{\text{MS}}}(M_Z)$ due to the brane localized kinetic terms is the same as long as unification happens before the theory hits the UV strong coupling scale.⁴ For our case, taking our central preferred value of $1/R_5 \simeq 4.4 \text{ TeV}$ and numerically evaluating the coefficients, we find the uncertainty on the $\sin^2 \theta_W$ prediction from 4D localized gauge kinetic terms is always less than 1%.

Thus we find that precision unification of the $SU(2)_L \times U(1)_Y$ interactions is relatively straightforward to accommodate in the MNSUSY framework with SSSB. We emphasize, of course, that this is not yet a full unification of SM gauge couplings, so we now turn to the issue of unifying $SU(3)_{\text{EW}}$ with $SU(3)_C$.

3 Possible $SU(6)$ unification of $SU(3)_{\text{EW}} \times SU(3)_C \times U(1)'$ in 6D

In this Section, we discuss the possible unification of $SU(3)_{\text{EW}}$ with $SU(3)_C$ into a 6D $N = 4$ supersymmetric $SU(6)$ gauge theory broken by orbifold bcs to a 5D $N = 2$ supersymmetric $SU(3)_{\text{EW}} \times SU(3)_C \times U(1)'$ gauge theory (related models were earlier studied in Refs.[46, 47]). We find that a *calculable* linear-power-law threshold effect plays a vital role.

⁴Unification can only be discussed if the unification scale is lower than the strong coupling scale of the theory derived from the unitarity constraint of Section 2.3, which we numerically find to be $\sim 150 \text{ TeV}$, in which case the precision of unification is set by $\sim N_i g_{4i}^2 / (16\pi^2)$ regardless of the size of the bulk.

3.1 The $SU(6)$ model

We first discuss the model being considered. We start with a pure 6D $N = 4$ supersymmetric $SU(6)$ gauge theory compactified down to 5D on an $S^1/(Z_2 \times Z'_2)$ orbifold of size πR_6 . In principle the bulk theory is tightly constrained by cancellation of 6D gauge and gravitational box anomalies but standard results [62–65] show that the $N = 4$ bulk theory is free of pure gauge and mixed gauge-gravity anomalies and only additional gauge singlet fields are needed to cancel the pure gravity box anomaly. Since these fields are gauge neutral, they do not affect gauge coupling unification. This theory is then compactified to 5D on a $S^1/(Z_2 \times Z'_2)$ orbifold in the 6th spatial direction z (see the left panel of Figure 1).⁵

In terms of 5D adjoint representation vector- and hyper-multiplets the bcs under the compactification are chosen to be

$$\begin{aligned} V(x^\mu, y, z) &= ZV(x^\mu, y, -z)Z^{-1} = Z'V(x^\mu, y, -z + 2\pi R_6)Z'^{-1} \\ \Phi(x^\mu, y, z) &= Z\Phi(x^\mu, y, -z)Z^{-1} = -Z'\Phi(x^\mu, y, -z + 2\pi R_6)Z'^{-1} \end{aligned} \quad (3.1)$$

where $Z, Z' = \text{diag}(1, 1, 1, -1, -1, -1), \text{diag}(1, 1, 1, 1, 1, 1)$. In other words the orbifold bcs for the bulk gauge multiplet are

$$V : \begin{pmatrix} (+, +)_{3 \times 3} & (-, +)_{3 \times 3} \\ (-, +)_{3 \times 3} & (+, +)_{3 \times 3} \end{pmatrix} \quad \Phi : \begin{pmatrix} (-, -)_{3 \times 3} & (+, -)_{3 \times 3} \\ (+, -)_{3 \times 3} & (-, -)_{3 \times 3} \end{pmatrix} \quad (3.2)$$

These bcs preserve $N = 2$ SUSY and $SU(3)_{\text{EW}} \times SU(3)_C \times U(1)'$ gauge symmetry in the low energy effective theory (correspondingly the zero mode spectrum is a $N = 2$ vector multiplet of $SU(3)_{\text{EW}} \times SU(3)_C \times U(1)'$). In more detail, the $z = \pi R_6$ fixed point preserves the $SU(6)$ gauge symmetry while breaking the $N = 4$ supersymmetry to $N = 2$, while the $z = 0$ fixed point is a $N = 2$ supersymmetric $SU(3)_{\text{EW}} \times SU(3)_C \times U(1)'$ gauge theory where the $SU(3)_{\text{EW}}$ -extended higgs and lepton hypermultiplets of Section 2 live – see the left panel of Figure 1. (The $U(1)'$ gauge symmetry is later broken at scale $1/R_5$. In Section 4, we will discuss the case where we identify this $U(1)'$ as the extended gauge structure needed to raise the higgs pole mass to its observed value.)

It will be important for our analysis that the *bulk* field content after the compactification has an accidental Z_2 symmetry that interchanges the two $SU(3)$ gauge groups, which is only broken by matter fields localized on the $z = 0$ brane. The feature that the three flavors of lepton fields and the higgs fields are the only source of breaking of the accidental Z_2 symmetry is a crucial ingredient that leads to precision predictions for unification of $SU(3)_{\text{EW}} \times SU(3)_C$.

We now turn to a discussion of the calculable differential power-law *threshold* effect that splits the $SU(3)_{\text{EW}} \times SU(3)_C$ gauge couplings at low energy starting from the $SU(6)$ -dominated unified value at high energies.

⁵It is important that this 6D to 5D orbifolding is not confused with the independent $S^1/(Z_2 \times Z'_2)$ orbifolding in the orthogonal 5th spatial direction, y , which has already been discussed in Section 2.

3.2 Calculable differential linear threshold effects

The counter-term for the bulk 6D gauge coupling has dimension $[\text{mass}]^2$ and therefore can in principle have linear and quadratic UV cut-off Λ dependence. However, in the present model, the bulk is exactly $SU(6)$ symmetric and $N = 4$ supersymmetric, and so there is no differential quadratic cut-off dependent threshold effect. In addition, in our model the KK mode spectrum of the 6D bulk fields respect an accidental Z_2 symmetry that interchanges the two $SU(3)$ gauge groups, and so there are no differential effects between the two $SU(3)$ gauge couplings due to 6D bulk supermultiplets at all.

In fact, the leading differential correction in our model is a linear threshold entirely due to the 5D fields localized on the $z = 0$ brane which do not respect the Z_2 symmetry between the two $SU(3)$ gauge groups. Remarkably these linear threshold effects can be calculated in a $N = 2$ supersymmetric gauge theory with the results of Intriligator, Morrison, and Seiberg [51, 52] as we now briefly explain (we closely follow the discussion of [56]).

Power-like threshold corrections to gauge couplings are a generic feature of higher dimensional gauge theory. Fortunately for us, these corrections are very constrained by the combined gauge symmetry and supersymmetry of the 5D theory under consideration. In the 4D superfield language of Section 2.1 the low energy description of the 5D SUSY theory, specifically the two-derivative terms that determine the low-energy gauge couplings, is fully characterized by a holomorphic pre-potential $\mathcal{F}(\Sigma)$ where $\Sigma = \Sigma^a T^a$ is the adjoint representation 4D chiral supermultiplet contained in the 5D vector supermultiplet:

$$\mathcal{L} = \frac{1}{2} \int d^4\theta \frac{\partial \mathcal{F}(\Sigma)}{\partial \Sigma^a} (\bar{\Sigma} e^{2V})^a + \frac{1}{2} \int d^2\theta \frac{\partial^2 \mathcal{F}(\Sigma)}{\partial \Sigma^a \partial \Sigma^b} W^a W^b + h.c. \quad (3.3)$$

(here the dependence of all the fields on the 5th-dimension coordinate is simply treated as an extra parameter). Vitally, the pre-potential is consistent with 5D gauge invariance only if it is at most cubic [51, 52]⁶ and this together with holomorphy and the restrictions of 5D Lorentz invariance strongly constrains the exact form of the quantum pre-potential. This feature is important because the unification with calculable power-like threshold corrections might be spoiled by non-calculable contributions from higher dimensional operators suppressed by $1/M_*$. In the case where the fundamental scale M_* is comparable to the unification scale,

⁶The $n > 3$ terms in the pre-potential will lead to, on the Coulomb branch, bosonic Lagrangian terms in the form of $\Sigma^{n-3} A_5 dA \wedge dA$ in terms of 4D gauge fields and scalars, or with 5D gauge fields $\Sigma^{n-3} A \wedge dA \wedge dA$. The $A \wedge dA \wedge dA$ piece is the 5D Chern-Simons term, which under gauge transformation become a total derivative. Therefore, only when $n \leq 3$ is $\Sigma^{n-3} A \wedge dA \wedge dA$ gauge invariant. Similar arguments apply to 6D $N = 2$ theories, with the result that $n \leq 4$, the corresponding Lorentz invariant Lagrangian term being $A_5 A_6 dA \wedge dA$. This term, upon compactification, might lead to corrections to the low energy effective theory at two derivative level. If both A_5 and A_6 are taken to be along the direction of $SU(3)_C \times SU(3)_{EW} \times U(1)'$, after compactification, the low energy 5D $SU(3)_C \times SU(3)_{EW} \times U(1)'$ gauge symmetry and $N = 2$ supersymmetry implies this term with dimension six is absent in the low energy theory. If A_5 and A_6 are both taken to be along the direction of $SU(6)/SU(3)_C \times SU(3)_{EW} \times U(1)'$, there will in principle be corrections at two derivative level to the gauge coupling. However, due to the accidental Z_2 symmetry of the bulk spectrum, these contributions are universal for the two $SU(3)$ and does not introduce non-calculable differential threshold effect.

these operators might totally destroy the unification prediction. However the form of the pre-potential, and thus all two-derivative terms is determined by the results of Refs. [51, 52, 56], and so the higher-dimension operators that might otherwise be dangerous for predictable unification are eliminated.

For example, the pre-potential at the classical level is

$$\mathcal{F}^{(cl)}(\Sigma) = \frac{1}{4g_{5,cl}^2} \delta_{ab} \Sigma^a \Sigma^b + \frac{c_{cl}}{48\pi^2} d_{abc} \Sigma^a \Sigma^b \Sigma^c, \quad (3.4)$$

with $d_{abc} \equiv \frac{1}{2} \text{Tr}(T_a \{T_b, T_c\})$, leading to the classical two derivative bosonic Lagrangian terms

$$\mathcal{L} \supset \frac{1}{4g_{5,cl}^2} \text{Tr} F^2 + \frac{c_{cl}}{16\pi^2} \text{Tr} \Sigma F^2, \quad (3.5)$$

where the second term is the 'classical' Chern-Simons (CS) term. After including quantum corrections, the pre-potential is constrained to have the *exact* form [51, 52, 56]

$$\mathcal{F}(\Sigma) = \mathcal{F}^{(cl)}(\Sigma) + \frac{1}{96\pi^2} \left(\sum_{\alpha} |\alpha_a \Sigma^a|^3 - \sum_f \sum_{\lambda} |\lambda_a \Sigma^a + m_f|^3 \right). \quad (3.6)$$

Here the sum runs over the roots of the unbroken gauge group α and the weights of the relevant matter representation λ , and the modulus signs only determine whether the cubic terms are to be multiplied by ± 1 , so $\mathcal{F}(\Sigma)$ remains locally holomorphic. The last two terms come from integrating out the gauge bosons in the broken directions and the heavy 5D matter fields with mass m_f . The corresponding effective 5D gauge coupling in the quantum theory is

$$\frac{1}{4g_{5,eff}(\tilde{\Sigma})^2} = \frac{1}{4g_{5,cl}^2} + \frac{c_{cl}}{16\pi^2} \tilde{\Sigma}^a + \frac{1}{16\pi^2} \left(\sum_{\alpha} \alpha_b^2 |\alpha_a \tilde{\Sigma}^a| - \sum_f \sum_{\lambda} \lambda_b^2 |\lambda_a \tilde{\Sigma}^a + m_f| \right), \quad (3.7)$$

where $\tilde{\Sigma}$ is the vev of the scalar component of the chiral adjoint field. It is important for us that this formula gives the exact dependence of the gauge coupling on the masses of the hypermultiplets m_f . Note that integrating out n_f 5D hypermultiplets with masses $m_f \rightarrow \pm\infty$ in the fundamental representation induces a shift in the effective CS term $c_{cl} \rightarrow c_{cl} \mp \frac{n_f}{2}$. This can be seen because for $m_f \gg |\Sigma^a|$, the sign of m_f defines the sign of the modulus of the final term, and expanding out the final term in linear order in $\tilde{\Sigma}$ effectively shifts $c_{eff} = c_{cl} \mp \frac{n_f}{2}$.

The additional term linear in the mass of the hypermultiplet m_f in the limit $m_f \rightarrow \pm\infty$ does not change the relative size of the gauge coupling between the strong coupling scale and the scale $1/R_5$. This is because at the strong coupling scale M_* (which we later take to be the unification scale M_{GUT}), it is the effective coupling $g_{5,eff}$ given by

$$\frac{1}{4g_{5,eff}^2} = \frac{1}{4g_{5,cl}^2} - \frac{1}{32\pi^2} \sum_f |m_f| \quad (3.8)$$

that goes to the strong coupling value. Thus the unitarity constraint on the model is not affected as heavy fermions are added to the theory, and can be calculated with knowledge of the low energy spectrum of the theory and the corresponding 'classical' CS-term coefficient of the low energy effective theory.

In our setup, the $SU(6)$ symmetry breaks down by boundary conditions to $SU(3)_C \times SU(3)_{EW} \times U(1)'$ with an accidental Z_2 symmetry interchanging $SU(3)_C$ and $SU(3)_{EW}$. We define our UV theory by demanding that the only breaking of the Z_2 symmetry in the 5D effective theory is a soft breaking due to the N=2 SUSY preserving masses of the brane localized hypermultiplets on the $z = 0$ brane.⁷

The theory below the Z_2 breaking scale M_0 has a 'classical' CS coefficient of $\delta c_d = 0$ and $+\frac{5}{2}$ for $SU(3)_C$ and $SU(3)_{EW}$, respectively. In the following, we will account for the Z_2 breaking linear threshold corrections in the low energy effective theory with the classical CS terms.

Since the only Z_2 breaking effect is the soft breaking due to the masses of the hypermultiplets (ignoring the gauge kinetic term on the $y = \pi R_5$ brane), the differential linear threshold effect between the 5D $SU(3)_{EW}$ and $SU(3)_C$ gauge couplings at the matching scale $1/R_6$ is

$$\frac{1}{g_{5,EW}^2(1/R_6)} - \frac{1}{g_{5,C}^2(1/R_6)} = \frac{5/2}{4\pi^2} M_0. \quad (3.9)$$

Here we use the fact that the Z_2 symmetry ensures that the gauge coupling and CS coefficients above the Z_2 breaking scale M_0 are the same.

It should be noted that there will also be logarithmic differential running between the two $SU(3)$'s, however, the logarithmic differential running and finite threshold corrections can be understood with 4d KK modes summation and therefore is already taken into account with methods introduced in Section 2.2.

3.3 The precision of $SU(6)$ unification and $\alpha_3(M_Z)$

The Z_2 symmetry on the $z = 0$ brane ensures that the $SU(6)$ breaking 5D gauge kinetic term is the same for $SU(3)_{EW}$ and $SU(3)_C$. However, there are Z_2 breaking 5D gauge kinetic terms on the $y = \pi R_5$ brane. The 5D gauge kinetic terms, like the 4D gauge kinetic term discussed in Section 2.4 will generate a non-calculable Z_2 breaking contribution to the gauge coupling at low energies, and therefore affect the precision of unification.

The unitarity constraint on the size of the sixth dimension and the precision of unification can be estimated with NDA [50]. Taking into account all powerlaw and logarithmic 1-loop effect, we find, with a fundamental scale at $M_* \sim 120$ TeV, the bound on the size of the sixth

⁷For example, we can preserve the Z_2 symmetry in the UV by having 5 pairs of triplet hypermultiplets under both $SU(3)_{EW}$ and $SU(3)_C$, but then softly break the Z_2 by taking masses 0 and M_0 for the 5 pairs of $SU(3)_{EW}$ and $\pm M_0$ for the 5 pairs of $SU(3)_C$. Then below M_0 , the heavy hypermultiplets are integrated out and the low lying $SU(3)_{EW}$ hypermultiplets $(\mathcal{L}, \mathcal{H}_{u,d})$ are the only Z_2 breaking matter content. Additional Z_2 symmetric matter content above M_0 will not change the differential threshold effect.

dimension is $M_* R_6 \leq 3$. Then, the size of the non-calculable threshold correction due to the (volume diluted) Z_2 breaking kinetic terms on the $y = \pi R_5$ brane is approximately 5% as follows from a standard strong coupling analysis. It should be noted the size of the non-calculable contributions does not directly translate to the precision of the prediction of gauge couplings at the electroweak scale. Combined with the estimate of the size of the operators localized on the 4D fixed point, the precision on the $\sin^2 \theta_W$ prediction is 2% (fixing α_3 and α_{EM}), while the precision of the prediction of α_3 is 7% (fixing $\sin^2 \theta_W$ and α_{EM}), signaling the unusual two step unification in this model.

4 $U(1)'$ and the higgs mass

4.1 $U(1)'$ symmetry breaking and anomalies

The $U(1)'$ from the broken $SU(6)$ symmetry can be used to lift the higgs pole mass to 126 GeV by choosing proper $U(1)'$ charge of the higgs and the fields ϕ_{\pm} , and 5D bcs of the $U(1)'$ gauge boson. The higgs $H_{u,d}$ must have opposite $U(1)'$ charge so that the brane localized divergent Fayet-Iliopoulos term vanishes [66–68]. The bcs for ϕ_{\pm} that lead to a non-decoupling D-term [57] is

$$\phi_{\pm} : \begin{pmatrix} (+, -) \\ (+, -) \\ (+, -) \end{pmatrix}, \quad \phi_{\pm}^c : \begin{pmatrix} (-, +) \\ (-, +) \\ (-, +) \end{pmatrix}, \quad \tilde{\phi}_{\pm} : \begin{pmatrix} (+, +) \\ (+, +) \\ (+, +) \end{pmatrix}, \quad \tilde{\phi}_{\pm}^c : \begin{pmatrix} (-, -) \\ (-, -) \\ (-, -) \end{pmatrix}. \quad (4.1)$$

The brane localized superpotential

$$W \supset \int dy dz \frac{\lambda}{M_*} X(\phi_+ \phi_- - v_{\phi}^3) \delta(y - \pi R_5) \delta(z), \quad (4.2)$$

with bcs in Eq.(4.1), generates both a vev for ϕ_{\pm} to break the $U(1)'$ gauge symmetry and an F-term for X around the size of the fifth dimension $1/R_5$ to get Standard Model Yukawa couplings [1]. The gauge coupling g' is normalized as $g'^2 = \frac{1}{3} g_c^2$ at the GUT scale. The differential linear threshold effect between the $U(1)'$ and $SU(3)_C$ dominates the differential logarithmic running, and the $U(1)'$ linear threshold beta function is $b_{U(1)'} = (33/4 + 2N_{\phi}^2)/3$, where N_{ϕ} is the $U(1)'$ charge of pairs of ϕ_{\pm} fields. Starting from a strongly coupled theory in the far UV, the low energy gauge coupling is approximately

$$g'^2 \simeq \frac{1}{3} \frac{4\pi^2}{b_{U(1)'}(M_* R_5)} \simeq 0.1 \quad (4.3)$$

for $N_{\phi} = 1$. Here, the Z_2 symmetry does not ensure a vanishing relative brane localized gauge kinetic term on the $z = 0$ brane, which generates the main uncertainty on the prediction of g' . The uncertainty in the size of g' and the ϕ_{\pm} field vev v_{ϕ} contribute a major uncertainty in the prediction of the $U(1)'$ gauge boson mass and the mass of the Higgs. With a more careful

treatment of the differential linear threshold effect and logarithmic running, the resulting $U(1)'$ gauge boson mass is

$$M_{U(1)'}^2 \simeq 2g'N_\phi v_{\phi_\pm}^{3/2} \left(\frac{1}{\pi R_5} \right)^{1/2} \approx (2.4 \pm 0.2 \text{ TeV})^2 \left(\frac{N_\phi}{1} \right) \left(\frac{v_\phi}{4.4 \text{ TeV}} \right) \left(\frac{g'}{0.26} \right) \quad (4.4)$$

The broken $U(1)'$ symmetry is not anomalous in 5 dimensions but has 4D anomalies with the $U(1)'$ charge assignment of the lepton \mathcal{L} and the right-handed quark u^c, d^c . The 4D anomalies of this broken symmetry suggest that localized fermions needed to be added on the $y = \pi R_5, z = 0$ or $y = 0, z = 0$ brane to UV complete the theory [69]. The mass of these states need to be lighter than

$$m \approx 4\pi \frac{M_{U(1)'}}{g'} \simeq 150 \text{ TeV} \quad (4.5)$$

It is not inconceivable that the mass of the localized states is close to the unification scale and does not significantly change the prediction of the $U(1)'$ gauge coupling at the electroweak scale. Indeed, the uncertainties on the predictions of the physical higgs mass due to the uncertainty of the masses of the heavy states are minor compared with other uncertainties.

4.2 Higgs mass from non-decoupling D-term

The contribution to the higgs quartic from the non-decoupling D-term can be written as [3]

$$m_h^2 \simeq 2 \left(\frac{1}{4} (g_1^2 + g_2^2) + 2\delta\lambda_{U(1)'} + \delta\lambda_{\text{stop LL}} + \delta\lambda_{\text{EW 1-loop}} + \dots \right) v_{\text{EW}}^2, \quad (4.6)$$

where $\delta\lambda_{U(1)'} = \frac{g'^2}{2} N_\phi^2 f(M_{U(1)'}, R)$ and $v_{\text{EW}} = 174 \text{ GeV}$. Here $f(M_{U(1)'}, R)$ is a function of the $U(1)'$ gauge boson mass and the size of the fifth dimension [1, 3] that determines the relative suppression on the $U(1)'$ contributions to the masses of the higgs boson due to decoupling. We include the all-orders leading-log contributions from the stop and fixed-order 1-loop leading-log contributions from the EW states to the quartic coupling of the Higgs⁸. The physical Higgs mass with the benchmark values (see table 2) for the mass parameters is

$$m_h \simeq 123.7_{-2.5}^{+2.7} \text{ GeV}, \quad (4.7)$$

with $U(1)'$ charge $N_\phi = 1$, and the mass of the top quark $m_t = 173.2 \pm 0.9 \text{ GeV}$. The comparison between the prediction of the physical higgs mass with the compactification scale $1/R_5 = 4.4 \text{ TeV}$ and the mass measured by ATLAS and CMS collaboration [70] is shown in Figure 3. The uncertainty on the higgs mass mainly comes from the theoretical uncertainty in the loop contributions from the stop and the experimental uncertainties on the top quark mass measurement ($\sim 2 \text{ GeV}$), and the uncertainties on the prediction of $U(1)'$ coupling g' and ϕ_\pm vev v_ϕ ($\lesssim 1.5 \text{ GeV}$). There are additional contributions to the higgs quartic coupling from electroweak box diagram with S_u and A^X , and \tilde{S}_u and λ^X , shifting the physical higgs mass by a negligible 0.05 GeV .

⁸See [3] for more details.

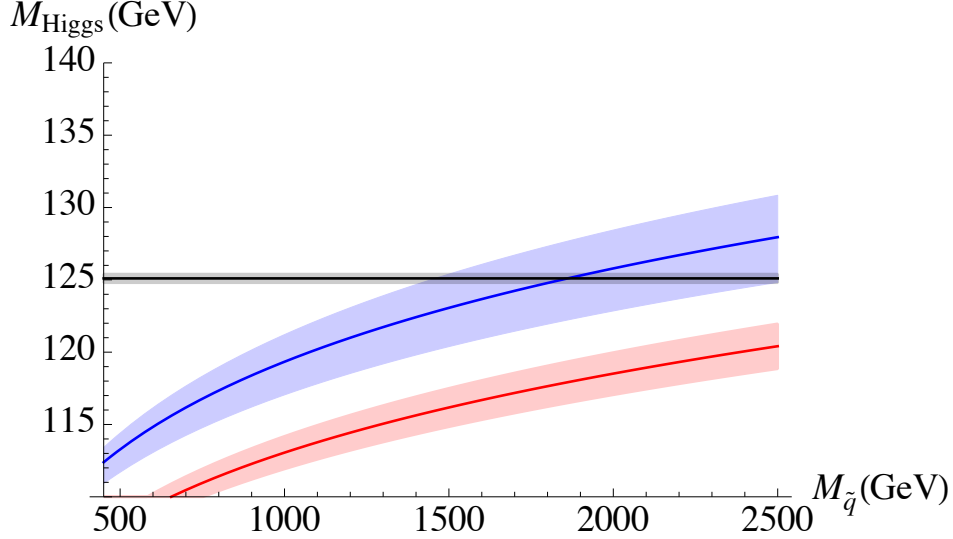


Figure 3. The physical higgs mass as a function of the squark (stop) masses with $1/R_5 = 4.4$ TeV. The blue curve is the prediction for the physical higgs mass with the contributions from the non-decoupling D-term of $U(1)'$ and the stop loop, with the light blue band the theoretical uncertainties on the prediction. The red curve is the prediction for physical higgs mass with the contributions from the the stop loop alone, with the light red band the theoretical uncertainties on the prediction. The black line and the corresponding band is the measured value [70].

5 Phenomenology of unified MNSUSY

The requirement that the theory unifies into a $SU(6)$ in six dimension favors certain relationships between the brane localized squark and heavy higgs masses, the fifth dimension size, $1/R_5$, and the zero mode gauginos λ^X and singlet higgs $S_{u,d}$ masses. In this Section, we discuss the current experimental probes of the light modes in the theory and their implications to the spectrum of the unified theory. We will focus on the case where the unified 6D theory is UV completed into a theory with a large gravitational bulk of dimension higher than 6, and assume that the light states in the theory will eventually decay into bulk LSPs [71].

5.1 Auto-concealment and limits on the squark masses

The brane localized squarks get masses from higher dimensional operators in the Kähler potential in Eq.(B.2). Since all three generation of squarks are on the brane, it is most natural to have their masses near degenerate. In this case, the first two generation squarks decay to jets plus bulk states while the stop decays to top quark plus bulk states. These decays are studied in [71] for the case where the stop decay is prompt. The decay width of the squarks is estimated to be [71]

$$\Gamma \simeq \frac{\Omega_d m_{\tilde{q}}^{d+3}}{8\pi(2\pi)^d M_*^{d+2}} \int_0^1 x^{d+1} (1-x^2)^2 dx, \quad (5.1)$$

where $m_{\tilde{q}}$ is the squark mass. With the benchmark value $m_{\tilde{t}} = 800$ GeV and $M_* = 120$ TeV, the squark and stop decay promptly when the number of large gravitational dimension $d = 3$, decay with a displaced vertex when $d = 4$ and are long-lived when $d \geq 5$. The experimental limit on the squark with three large gravitational dimension is around 450 GeV [71]. When the first two generation squarks have significant different mass compared with the stop, to be more specific, $|\delta m| = |m_{\tilde{t}} - m_{\tilde{q}_{1,2}}| > m_t$, the constraint is more stringent due to signatures from multiple top quarks from the cascade decay. Throughout the paper, we assume $m_{\tilde{t}} \simeq m_{\tilde{q}_{1,2}}$.

5.2 Mass limits on the zero mode gauginos and singlet higgs

The masses of the singlet higgs and gauginos come from the superpotential and Kähler potential terms in Eqs.(B.2) and (B.3) respectively. The mass of the gauginos λ^X is ~ 2 TeV when the size of the fifth dimension $1/R_5 \sim 4$ TeV, while the mass of the singlet higgs $S_{u,d}^c$ is less than ~ 500 GeV.

These light charged particles are potentially observable in the LHC. The singlet higgs $S_{u,d}$ couples to the SM fields only through $SU(3)_{EW}$ gauge couplings. The production cross section of the singlet higgs $S_{u,d}$ is twice the production cross section of the right handed stau with all gauginos decoupled. These singlet higgs decay through off-shell gauge boson A^X to $H_{u,d}$ and dilepton as shown in Figure 4. A single event from $S_{u,d}$ decay has $\sim 20\%$ chance to have four or more first two generation leptons with significant missing energy. The two leptons, l and e^c , will be same-sign same-flavor dilepton if A^X is doubly charged. The chance of having two same-sign same-flavor first two generation leptons, jets and significant missing energy in the decay of $S_{u,d}$ is also approximately 20%. The stringent constraint on their masses comes from multi-lepton [72, 73] and same-sign dilepton [74, 75] searches, and the limit can be estimated to be 350 GeV. These singlet higgs, acquiring their masses through similar higher dimensional operators as the heavy higgses in MSSM, is most likely to get a mass comparable to the heavy higgses H_d . The coefficient $c_{H_{u,d}}$ and $c_{S_{u,d}}$, however, are not related by $SU(3)_{EW}$ gauge symmetry and therefore it is not inconceivable that the doublet and singlet has a mass splitting large enough for the decay to be prompt, for example, contributions from the electroweak symmetry breaking. The collider signature of the case where S_d is near degenerate with H_d depends highly on how H_d decays. We will not attempt to provide an in-depth study of that scenario.

The gaugino λ^X forms an $SU(2)_L$ doublet containing a pair of doubly charged fermion ($\lambda^{\pm\pm}$) and a pair of singly charged fermions (λ^\pm) under $U(1)_{EM}$. These λ^X gauginos get produced with similar cross sections as the MSSM electro-weakinos [76–79] through both charged current and neutral current processes as shown in Figure 5.

The splitting between the two fermions, $\lambda^{\pm\pm}$ and λ^\pm , is equal to the usual splitting between the charged and neutral winos in anomaly mediated supersymmetry breaking (AMSB) scenarios [80, 81], with one loop splitting [82, 83] between $\lambda^{\pm\pm}$ and λ^\pm being ≈ 1.01 GeV. This splitting leads to a prompt decay of $\lambda^{\pm\pm}$ to λ^\pm and a charged pion. The singly charged gauginos λ^\pm are the lightest supersymmetric particle along the broken directions of $SU(3)_{EW}$ and therefore decay to the true LSP states in the gravitational bulk. The dominant channel

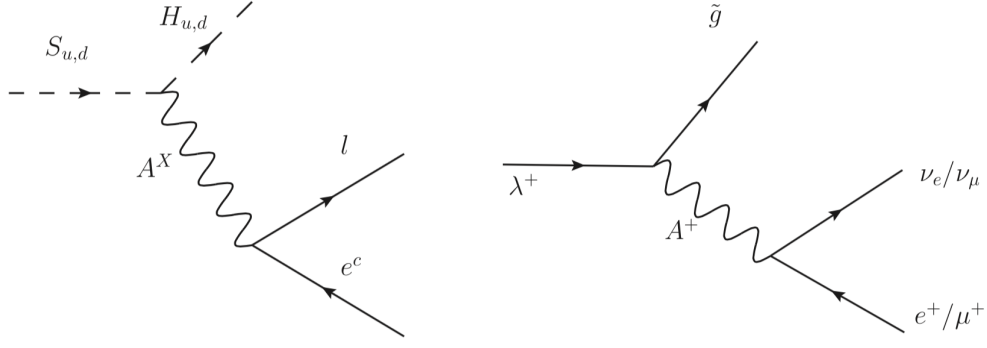


Figure 4. *Left panel:* The singlet higgs $S_{u,d}$ decay through heavy gauge boson to lepton pair. *Right panel:* The displaced decay of singly charged gaugino λ^+ to charged leptons and missing energy.

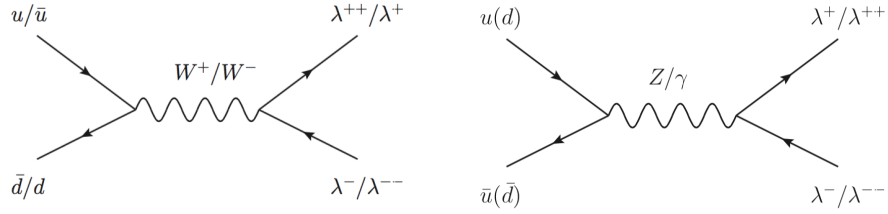


Figure 5. The leading LHC production processes of the singly and doubly charged gauginos λ^X .

for them to decay is through processes shown in Figure 4. The decay width of these particles is [71]

$$\Gamma \sim \frac{g_{SU(3)EW}^2}{(4\pi)^3} \frac{m_{\lambda^X}^5}{1/2R^4} \frac{\Omega_d m_{\lambda^X}^{d+2}}{(2\pi)^d M_*^{d+2}} \int_0^1 x^{d+1} (1-x^2)^2 dx, \quad (5.2)$$

The lifetime of λ^\pm is approximately 1 – 50 ns for $d = 3$, while λ^\pm become relatively collider-stable for $d > 3$. Such collider-stable λ^+ is constrained by searches for stable charged particles in LHC [84, 85]. Recasting the bound on chargino pair production suggest gauginos, λ^X , with masses smaller than 1 TeV are excluded. For $d = 3$, the collider signature is a mixture of kinked vertices [86, 87] and long-lived charged particles depending on the mass of the gaugino λ^X and the fundamental scale $1/M_*$. Searches in CMS [85, 88, 89] and ATLAS [84, 90] limits the mass of the charginos to be heavier than 750 GeV for λ^\pm with lifetime longer than 1 ns.

The decay of these gauginos λ^X and singlet higgs $S_{u,d}$ produces very distinct signatures in the LHC including kinked charged tracks, high p_T same-sign same-flavor di-lepton pair with significant missing energy. Thus we believe that LHC 14 has a good coverage over most of the natural parameter space of the model that explains both EW unification and the higgs mass. A detailed analysis of possible search optimizations is beyond the scope of this paper.

5.3 $U(1)'$ gauge boson

The $U(1)'$ gauge boson is produced at the LHC with cross section comparable to that of standard Z' gauge bosons. Depending on the size of the fifth dimension $1/R_5$, their masses and couplings to the Standard Model quarks and leptons vary from 2.5 TeV to 3.5 TeV and 0.2 to 0.3, respectively. Since the $U(1)'$ gauge coupling is much smaller than the $SU(3)_{EW}$ gauge coupling at $1/R_5$, the cross section for production of the $U(1)'$ gauge boson is approximately 40% to 80% (for $g' = 0.2$ and 0.3, respectively) of the production cross section of a Z'_{SM} with the $SU(2)_L$ gauge coupling. The right-handed quarks have twice the $U(1)'$ charge compared with the leptons while the left-handed quarks are not charged under $U(1)'$. As a result, the $U(1)'$ gauge boson has 16/27 branching ratio to decay into first two generation quarks and 2/81 branching ratio to decay into either electron or muon pairs. Reinterpreting the constraints from searches for di-jet resonance and di-lepton resonance gives a conservative limit on Z' mass around 2.3(2.5) TeV [91–94] for $g' = 0.2(0.3)$. Current LHC searches for heavy resonances do not yet constrain the most interesting parts of parameter space of our model, but future searches for di-lepton resonances should provide a useful reach.

5.4 Proton decay and neutron anti-neutron oscillation

Proton decay is a generic prediction of grand unified theories [95], and the non-observation of proton decay puts strong constraints on the form of the unified theory, the constraint being especially severe when the quantum gravity scale is low [96]. Given that the fundamental scale is bounded to be around 100 TeV in our model due to unitarity considerations, operators with dimension 4,5,6 are all dangerous and may lead to too fast proton decay. However, the $U(1)'$ gauge symmetry can be used to suppress dangerous proton decay operators: With the charge assignment for the $U(1)'$ in Table 3, the proton decay operator is forbidden due to the broken $U(1)'$ gauge symmetry. If the field ϕ_{\pm} is integer charged under $U(1)'$, the dangerous proton decay operator vanishes since L, e^c are the only half-integer charged fields under the $U(1)'$ symmetry.

Moreover, the neutron anti-neutron oscillation operators [97, 98] that are unique in SUSY theories, *i.e.*, the superpotential operators, can be eliminated by unbroken R-symmetry on the $z = 0, y = \pi R_5$ brane. Kähler operators localized on $z = 0, y = \pi R_5$ brane might lead to neutron anti-neutron oscillation if the $U(1)'$ charge of ϕ_+ is $N_{\phi} = 1, 2$. For example, if $N_{\phi} = 1$

$$K \supset \int d^4\theta \delta(y - \pi R_5) \delta(z) \frac{c_{12}}{M_*^8} QQQQ d^{c\dagger} d^{c\dagger} X^{\dagger} \phi_+ \phi_+ \quad (5.3)$$

Once supersymmetry is broken, this leads to the operator

$$\mathcal{L} \supset \frac{c_{12}}{M_*^8} QQQ\tilde{Q}\tilde{Q} d^{c\dagger} d^{c\dagger} F_X^{\dagger} v_{\phi_+}^3 + \dots \quad (5.4)$$

Neutron anti-neutron oscillation happens when the squark loops are completed with gauginos and leads to the dimension 9 operator similar to that of the Standard Model. Such an operator

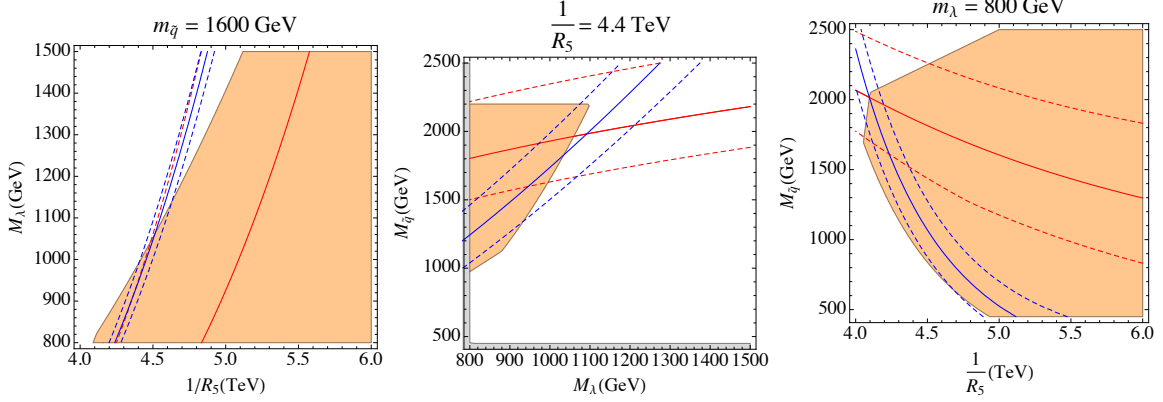


Figure 6. These figures show the allowed ranges of the gaugino mass, m_λ , the stop mass, $m_{\tilde{q}}$, and the compactification scale, $1/R_5$, that lead to the experimentally measured physical higgs mass and α_3 . The light grey regions are the experimentally excluded regions from various collider constraints on the low-lying charged particles, the light orange region is the allowed region that satisfies the unitarity constraint and gives large enough higher dimensional operators to generate the right λ^X -gaugino mass and stop masses smaller than $1/2R_5$. The red (red-dashed) line marks the central value (contours) of higgs mass that agrees with experiment and the blue (blue-dashed) line marks the central value (contours) of observed α_s taking into account the various theoretical and experimental uncertainties. *Left panel:* The relationship between the gaugino mass m_{λ^X} and the compactification scale $1/R_5$ fixing $m_{\tilde{q}} = 1600$ GeV, the missing red-dashed line is further to the right of the plot. *Middle panel:* The relationship between the gaugino mass m_{λ^X} and the squark masses $m_{\tilde{q}}$ fixing $1/R_5 = 4.4$ TeV. *Right panel:* The relationship between the squark masses $m_{\tilde{q}}$ and the compactification scale $1/R_5$ fixing the gaugino mass to be $m_{\lambda^X} = 800$ GeV.

produces neutron anti-neutron oscillation at the rate

$$\delta m \simeq 10^{-41} \text{ GeV} \left(\frac{c_{12}}{1} \right) \left(\frac{120 \text{ TeV}}{M_*} \right)^8 \left(\frac{v_{\phi_+}}{5 \text{ TeV}} \right)^3 \left(\frac{\Lambda_{QCD}}{200 \text{ MeV}} \right)^6 \quad (5.5)$$

and given current bound from the ILL reactor experiment [99], and comparable bounds from nucleon decay experiments [100–103], of $\delta m = 6 \times 10^{-33}$ GeV, the theory is safe from neutron anti-neutron oscillation constraints.

Finally, the higher dimensional operators that lead to proton decay and neutron anti-neutron oscillation could be generated by weak quantum gravity effect alone, but they can receive extra, exponential, suppression in some string theories into which the unified theory UV completes. We note that in our setup where the quarks are only charged under the SM group and the leptons only charged under $SU(3)_C \times SU(3)_{EW}$, the calculable gauge boson or gaugino mediated proton decay is totally absent. Therefore, though additional assumptions on the UV completion might in some cases be required, it is still conceivable that the minimal model shown in Figure 1 does not lead to proton decay.

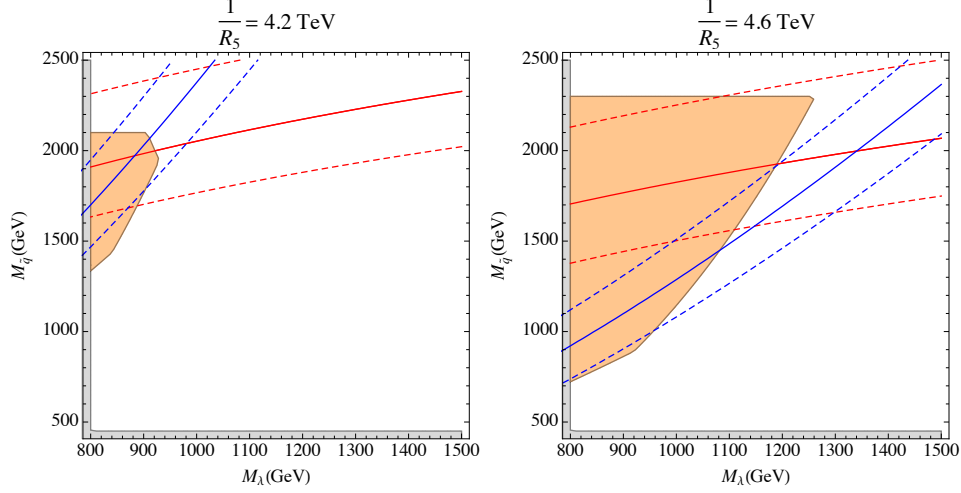


Figure 7. Similar to Figure 6. *Left panel:* The relationship between the gaugino λ^X mass and the squark masses $m_{\tilde{q}}$ taking $1/R_5 = 4.2$ TeV. *Right panel:* The relationship between the gaugino λ^X mass and the squark masses $m_{\tilde{q}}$ taking $1/R_5 = 4.6$ TeV.

5.5 Summary and unification favored particle spectrum

The unification scenario predicts new particles with distinct collider signatures that can be looked for in the next LHC run. Unification (assuming no additional field content below M_{GUT} that adds to the $SU(6)$ breaking by orbifold bcs and brane localized matter) predicts a relationship between the masses of the light modes that are of experimental interest. In Figure 6, we demonstrate the mass relationship between the gaugino mass, m_{λ^X} , the size of the fifth dimension, $1/R_5$, and the squark masses, $m_{\tilde{q}}$, keeping the heavy higgs masses $m_{H_{u,d}}, m_{S_{u,d}}$ fixed at 400 GeV⁹. These plots provide a guide to experimentally search for signatures of a low scale unified model with a natural spectrum.

The unification of gauge couplings and the correct prediction for the higgs mass requires that the compactification scale to be $\frac{1}{R_5} \gtrsim 2.2$ TeV so that the zero mode λ^X -gaugino and the squarks masses required are not experimentally excluded (here taking into account auto-concealment [71]) and also that the Z' mass is not excluded by searches for di-jet resonances at LHC, shown in Figure 7. As the compactification scale increase, unification and the higgs mass prefers smaller squark masses and larger gaugino masses and when $1/R_5$ approaches 5.8 TeV, the size of the higher dimensional operators in the theory is no longer large enough to generate the zero mode gaugino mass needed to have unification, shown in Figure 7.

To conclude, unification and the requirement to get the higgs mass from non-decoupling D-term of $U(1)'$ sets stringent constraint on the viable parameter space that produces precision unification in six dimensions. We are aware of new vector-like pairs of leptons may also generate one-loop contribute to the physical higgs mass in the MNSUSY setup [3]. These

⁹The effect of the change of the heavy higgs masses is small because of both the low multiplicity of the states and the small range of masses within which they are both experimentally and theoretically allowed

particles might change the relative running between the $SU(3)_C$ and $SU(3)_{EW}$, we will not discuss that case in this paper.

6 Conclusion and Remarks

In this paper we have constructed and studied a unified version of the MNSUSY theory. At the first stage of unification $SU(2)_L \times U(1)_Y$ is unified into a 5D $N = 2$ supersymmetric $SU(3)_{EW}$ gauge theory on an orbifold line segment of size πR_5 . Despite the higher-dimensional nature of the theory above the energy scale $1/R_5 \sim 4.4$ TeV (the energy scale at which Scherk-Schwarz supersymmetry breaking also takes place), the differential running between the $SU(2)_L$ and $U(1)_Y$ gauge couplings is *calculable and logarithmic* and gives the small corrections necessary to move the $SU(3)_{EW}$ prediction of $\sin^2 \theta_W = 1/4$ to the experimentally determined LEP value. Specifically, in Section 2 we show that $\sin^2 \theta_W(M_Z) = 0.2312$ is correctly predicted to within 2% uncertainty in the region where electroweak symmetry breaking is better than $\sim 10\%$ tuned. We stress that, in the context of MNSUSY (in contrast to some previous uses of $SU(3)_{EW}$ [41–44]) this unified prediction is not ‘tuned’ to reproduce the right value $\sin^2 \theta_W(M_Z)$ by freely varying some continuous parameter of the theory. In particular, the size, R_5 , of the 5th dimension is *independently fixed* by the requirement that Scherk-Schwarz supersymmetry breaking leads to successful electroweak symmetry breaking without excessive fine-tuning. Moreover, the parametrically large size of the 5th dimension compared with the fundamental scale of the theory guarantees that any brane localized operators that do not respect the bulk symmetry are volume suppressed, and do not spoil the precision prediction of $\sin^2 \theta_W(M_Z)$.

At the second stage of unification, $SU(3)_{EW} \times SU(3)_C$ (as well as an additional $U(1)'$ necessary for lifting the physical Higgs state to its observed mass – see Section 4) unifies into a six-dimensional $N = 4$ supersymmetric $SU(6)$ gauge theory defined on a $S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)$ rectangular orbifold of extra-dimensional size $\pi^2 R_5 R_6$. Both logarithmic running and *linear* threshold corrections differentially correcting the gauge couplings. While, as we argue in Section 3, the enlarged gauge symmetry and supersymmetry in the 6D bulk ensure that the both the differential logarithmic running and linear threshold corrections are calculable and various higher dimensional operators that spoil precision unification are not present in the bulk unified theory, the corrections due to $SU(6)$ -violating, and especially Z_2 violating, brane-localized operators are not so suppressed as in the $SU(3)_{EW}$ case. (Due to the fact that $1/R_6 \sim 40$ TeV is substantially closer to the fundamental scale of the underlying UV theory.) Correspondingly, we find the derived value of $\alpha_s(M_Z)$ is known to only $\pm 7\%$, reflecting the unusual two-step unification.

As discussed in Section 5, the unification scenario predicts new particles with distinct collider signatures that can be looked for in present and future LHC runs. The allowed mass range for these light modes and the compactification scale are summarized in Table 2. The squark and λ^X -gaugino masses that lead to precision unification are all within reach of the LHC, producing distinct signatures in their decays.

	$m_{\tilde{q}}$ (GeV)	m_{λ^X} (GeV)	$m_{S_{u,d}}$ (GeV)	$m_{Z'}$ (TeV)	$\frac{1}{R_5}$ (TeV)	$\frac{1}{R_6}$ (TeV)
BM	1600	800	400	2.5	4.4	~ 40
Range	(1400, 2300)	(800, 1200)	(400, 500)	(2.3, 3.5)	(4.1, 4.7)	

Table 2. Benchmark (BM) value of various mass and energy scales in the theory leading to full unification in 6D at ~ 120 TeV with the correct physical higgs mass, and with better than 10% tuning of EWSB. The mass range of the low lying states that satisfies all current experimental (roman numbers) and theoretical and fine-tuning constraints (italic numbers) is also provided. In addition to requiring low fine-tuning, here we also assume higher dimensional operators can be present with at most NDA value coefficients. Thus masses of particles exceeding these ranges are possible if unexplained tuning is allowed.

Finally, we remark that unification of $SU(3)_{EW}$ and $SU(3)_C$ via a differential *linear* threshold effect is a necessity because of the low fundamental scale required by perturbative unitarity considerations. However, following the work of Seiberg [51], progress [51–54] has been made towards the construction of exact finite theories in both five and six dimensions with non-trivial interacting fixed points. These studies might open up new possibilities for UV completions of MNSUSY into a theory with a non-trivial interacting fixed point, and, in that case, it is possible that the complete unification of MNSUSY occurs via purely logarithmic running, as in the MSSM, though of a highly unusual extra-dimensional form.

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A Renormalization scheme and scheme dependent constants

In this paper, the logarithmic running of the gauge couplings and the crossing of multiple mass thresholds is treated in the \overline{DR} scheme, in which no finite threshold corrections are present [104]. On the other hand, the gauge couplings are usually given in the \overline{MS} scheme [105]

$$\begin{aligned}
\alpha_{EM}^{\overline{MS}}(M_Z)^{-1} &= 127.944 \pm 0.014 \\
\sin^2 \theta_W^{\overline{MS}}(M_Z) &= 0.23116 \pm 0.00012 \\
\alpha_3^{\overline{MS}}(M_Z) &= 0.118 \pm 0.003
\end{aligned} \tag{A.1}$$

Field	Q	u^c	d^c	L	e^c	H_u	H_d	H_u^c	H_d^c	X	S	ϕ_+	ϕ_-
$U(1)_R$	1	1	1	1	1	0	2	2	0	2	2	0	0
$U(1)'$	0	-1	1	1/2	1/2	1	-1	-1	1	0	0	N_ϕ	$-N_\phi$

Table 3. $U(1)_R$ and $U(1)'$ charge assignments of the bulk and brane localized fields to be consistent with the superpotential and Kähler potential terms in the paper

which translate to the $SU(3)_{EW} \times SU(3)_C$ normalization to be

$$\begin{aligned}
\alpha_1^{\overline{MS}}(M_Z)^{-1} &= 32.790 \pm 0.006 \\
\alpha_2^{\overline{MS}}(M_Z)^{-1} &= 29.576 \pm 0.016 \\
\alpha_3^{\overline{MS}}(M_Z) &= 0.118 \pm 0.003
\end{aligned} \tag{A.2}$$

The matching between the \overline{MS} scheme and \overline{DR} scheme at M_Z is determined purely by the low energy spectrum below M_Z . Specifically (utilizing the results of [104, 106, 107])

$$\begin{aligned}
\alpha_1^{\overline{DR}}(M_Z)^{-1} &= \alpha_1^{\overline{MS}}(M_Z)^{-1} \\
\alpha_2^{\overline{DR}}(M_Z)^{-1} &= \alpha_2^{\overline{MS}}(M_Z)^{-1} + \frac{1}{6\pi} \\
\alpha_3^{\overline{DR}}(M_Z)^{-1} &= \alpha_3^{\overline{MS}}(M_Z)^{-1} + \frac{3}{12\pi}
\end{aligned} \tag{A.3}$$

Finally, the top quark mass in the \overline{MS} scheme is taken to be 160_{-4}^{+5} GeV [3] in the calculation of the physical higgs mass and coupling evolution.

B Generating masses for the exotic states

Operators on the $z = 0$ brane are required to generate Yukawa couplings and squark masses. In addition two brane-localized SM singlets X and S are needed to raise the masses of the exotic zero modes of the gaugino λ^X and singlet higgses $S_{u,d}$. Specifically, in terms of the 4d superpotential and Kähler potential on $y = \pi R_5, z = 0$ brane:

$$\begin{aligned}
W &\supset \int dy dz \delta(y - \pi R_5) \delta(z) \frac{y_u}{M_*^{1/2}} H_u Q u^c \\
K &\supset \int dy dz \delta(y - \pi R_5) \delta(z) \left\{ \frac{y_d}{M_*^{3/2}} \left(H_u^\dagger X^\dagger Q d^c \right) + \frac{y_L}{M_*^{5/2}} \left(H_u^\dagger X^\dagger L e^c \right) \right\}
\end{aligned} \tag{B.1}$$

where X develops a SUSY breaking F-term at $O(1/R_5)$. The bulk higgses and brane localized squarks get masses through the higher dimensional operators in the Kähler potential on the

$y = \pi R_5, z = 0$ brane

$$K \supset \int dydz \delta(y - \pi R_5) \delta(z) \left\{ \frac{1}{M_*^2} \left(c_Q X^\dagger X Q^\dagger Q + c_u X^\dagger X u^{c\dagger} u^c + c_d X^\dagger X d^{c\dagger} d^c \right) + \frac{1}{M_*^3} \left(c_{H_u} X^\dagger X H_u^\dagger H_u + c_{H_d} X^\dagger X H_d^\dagger H_d + c_{S_u} X^\dagger X S_u^\dagger S_u + c_{S_d} X^\dagger X S_d^\dagger S_d \right) \right\} \quad (\text{B.2})$$

These operators also provide both tree level and one loop (through the stop loop) contributions to the higgs soft-mass-squared and triggers electroweak symmetry breaking [1, 3]. The $U(1)_R$ symmetry is explicitly broken by higher-dimensional superpotential operators on the $y = 0, z = 0$ brane

$$W \supset \int dydz \delta(y) \delta(z) \frac{c_{\lambda^X}}{M_*} S W^\alpha W_\alpha, \quad (\text{B.3})$$

where S develops a SUSY breaking F-term at less than $1/2R_5$ to generate the required gaugino masses, with W^α defined by the gauge kinetic term in 6D,

$$W \supset \int dydz \frac{1}{g_6^2} W^\alpha W_\alpha. \quad (\text{B.4})$$

There are also other $U(1)_R$ symmetry breaking higher-dimensional operators that generates a $B\mu$ -term for the Higgs, and a much suppressed A-term due to locality. The structure suggests that the mass of the squarks and gauginos are likely comparable while the higgs ($\mathcal{H}_{u,d}$) are a factor of $\frac{1}{(\pi M_* R_5)^{1/2}}$ smaller. The Lagrangian of the theory to reproduce the low energy mass spectrum of the SM matter can be written down on the $SU(3)_{\text{EW}}$ broken brane as

$$\mathcal{L} = \int dydz \delta(y - \pi R_5) \delta(z) \left(y_u H_u Q u^c + y_d H_u^\dagger Q d^c + y_L H_u^\dagger L e^c + M_{S_{u,d}}^2 S_{u,d}^\dagger S_{u,d} + M_{H_{u,d}}^2 H_{u,d}^\dagger H_{u,d} \right) + \int dydz \delta(y) \delta(z) M_{\lambda^X} \lambda^X \lambda^X \dots \quad (\text{B.5})$$

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